UDC 532.593

ON STEADY COMPOSITE CAPILLARY-GRAVITATIONAL WAVES OF FINITE AMPLITUDE

PMM Vol. 39, №2, 1975, pp. 298-305

Ia. I. SEKERZH-ZEN'KOVICH
(Moscow)

(Received August 7, 1974)

Steady waves of finite amplitude induced by pressure periodically distributed along the fluid surface and disappearing when this pressure becomes constant are called here induced waves. Steady waves of finite amplitude occurring under constant surface pressure at particular velocities of the stream are called free waves. Induced capillary-gravitational waves at the surface of an infinitely deep fluid were investigated in [1, 2], while similar free waves were analyzed in [3, 4].

The possibility of concurrent existence of both kinds of such capillary-gravitational waves of small but finite amplitude at some particular velocity of the stream at infinite depth is considered below. These waves are called composite waves. When the varying component of pressure distributed over the surface becomes identically zero, these waves do not disappear but are transformed into free waves.

The problem is considered in a rigorous formulation and reduces to the solution of three nonlinear equations one of which is integral and the remaining two transcendental. The pressure at the surface is defined by an infinite trigonometric series whose coefficients are proportional to integral powers of some dimensionless small parameter, which are by two units higher than their index number.

The theorem of existence and uniqueness of solution is established, and the method of its proof is indicated. Derivation of solution in the form of series in powers of the small parameter mentioned above with any approximation is described. The first three approximations are completely calculated. An approximate equation of the wave profile is presented. Purely gravitational composite waves were considered in [5].

1. Statement of problem and derivation of basic equations. Let us consider a plane-parallel steady motion of a perfect incompressible heavy fluid bounded only from above by a free surface subjected to pressure $p_0 = p_0' + p_0(x)$, where $p_0' = \text{const}$, and $p_0(x)$ is a specified periodic function of the horizontal coordinate x. The stream is assumed to flow from left to right at specified constant velocity c at infinite depth. As already indicated, induced waves will occur at any velocity c whenever the term $p_0(x)$ is present. In the absence of $p_0(x)$ free waves appear at certain particular values of c. It is assumed here that the pressure at the free surface contains both of these terms. The free surface represented in coordinates attached to a wave progressing at velocity c, has the form of a stationary periodic wave. We seek waves which do not disappear but for $p_0(x) \equiv 0$ and specific velocities c are transformed into free waves. As already indicated the latter are called composite waves.

Let the unknown composite wave and pressure $p_0(x)$ be symmetric about the vertical drawn through the crest. We superpose the y-axis of an orthogonal system of coordinates xy on the axis of symmetry and direct that axis upward. We locate the coordinate origin O at the point of intersection of the y-axis with the free surface and direct the x-axis to the right.

We take the plane of flow xy as the plane of the complex variable $z = x + iy$. Let φ denote the potential of velocities, ψ the stream function, $w = \varphi + i\psi$ the complex potential of velocities, and U and V denote the projections of the velocity vector $\mathbf{q}$ on the coordinate axes. We then have

$$dw \,/\, dz = -U + iV, \quad U = -\partial\varphi \,/\, \partial x, \quad V = -\partial\varphi \,/\, \partial y$$

To derive the basic equations of this problem we first conformally map the region occupied by a single wave and represented by a vertical infinite half-band bounded from above by a wave-like curve on the half-band $0 \leqslant \varphi \leqslant c\lambda$, $0 \leqslant \psi \leqslant \infty$ in the w-plane, and then map this half-band into the interior of a unit circle whose center in the plane $u = u_1 + iu_2$ lies at the coordinate origin. It is assumed that the wavelength λ corresponds to the period of function $p_0(x)$. The mapping is carried out by formula

$$w = \frac{\lambda c}{2\pi i} \ln u \tag{1.1}$$

which transforms the wave profile into the circumference of the unit circle with a slit along the radius $\arg u = 0$. Mapping of circle $|u| \leqslant 1$ in the region of a single wave in the z-plane is determined by formula

$$\frac{dz}{du} = -\frac{\lambda}{2\pi i} \frac{e^{i\omega(u)}}{u}, \qquad \omega(u) = \Phi + i\tau \tag{1.2}$$

Since function $\omega(u)$ is holomorphic, it can be represented by a Taylor series inside the circle. Owing to the symmetry of the wave, the coefficients of this series must be real.

From (1.1) and (1.2) we obtain

$$dw \,/\, dz = -ce^{\tau - i\Phi}$$

This implies that throughout the stream function Φ is equal to the angle between the velocity vector $\mathbf{q}$ and the x-axis and that

$$q = |\mathbf{q}| = ce^{\tau} \tag{1.3}$$

Note that in the expansion of $\omega(u)$ the free term $A_0 = 0$, since at infinity the velocity of the stream is equal c and is parallel to the x-axis.

For $u = e^{i\theta}$ (θ is the angle of the position radius to the u_1-axis) (1.2) yields a differential equation; separating in it real and imaginary terms and integrating, we obtain for the wave profile the parametric equation

$$x = -\frac{\lambda}{2\pi} \int_0^\theta e^{-\tau(\eta)} \cos \Phi(\eta)\, d\eta, \quad y = -\frac{\lambda}{2\pi} \int_0^\theta e^{-\tau(\eta)} \sin \Phi(\eta)\, d\eta \tag{1.4}$$

where

$$\tau(\eta) = \tau(1, \eta), \quad \Phi(\eta) = \Phi(1, \eta)$$

It follows from (1.4) that for solving the problem it is necessary to determine besides $\Phi(\theta)$ also $\tau(\theta)$. The expansion of function $\omega(u)$ shows that these functions can be represented by the following trigonometric series

$$-\tau(\theta) = \sum_{n=1}^{\infty} A_n \cos n\theta, \qquad \Phi(\theta) = \sum_{n=1}^{\infty} A_n \sin n\theta \tag{1.5}$$

where in accordance with the previous statement $A_0 = 0$. Expansions (1.5) satisfy the condition of symmetry of the unknown wave about the vertical passing through its crest.

We represent the boundary condition along the surface by the Bernoulli integral

$$p / \rho = C - gy - {}^1/_2 q^2 \qquad (1.6)$$

where C is a constant, g is the acceleration of gravity, and ρ is the density. At the free surface the difference of pressures is balanced by the resultant of surface tension forces. By the Laplace law for these forces we have

$$p - p_0 = \pm \mu / R \qquad (1.7)$$

where p is the pressure from the side of the fluid, $p_0 = p_0' + p_0(x)$ is the pressure from outside the free surface, μ is the capillary constant, and R is the radius of curvature at points of the surface. Expressing the curvature in terms of $d\Phi / d\theta$, we obtain

$$p - p_0 = \frac{2\pi\mu}{\lambda c} q \frac{d\Phi}{d\theta} \qquad (1.8)$$

Substituting expression (1.8) for p into (1.6) and taking into account (1.3), we obtain

$$\frac{d\Phi}{d\theta} = \nu \left[\delta e^{-\tau} - e^{\tau} - \frac{2\pi}{\lambda} \varkappa y e^{-\tau} - p_0^*(x) e^{-\tau} \right] \qquad (1.9)$$

$$\nu = \frac{\lambda c^2 \rho}{4\pi\mu}, \quad \delta = \frac{2(C\rho - p_0')}{\rho c^2}, \quad \varkappa = \frac{g\lambda}{\pi c^2}, \quad p_0^*(x) = \frac{2p_0(x)}{\rho c^2} \qquad (1.10)$$

where x and y which are functions of θ are determined by formulas (1.4). Taking into account the formula for y and separating in the right-hand part of (1.9) terms which are linear with respect to Φ and τ, we obtain

$$\frac{d\Phi}{d\theta} = \nu \left\{ \delta - (\delta + 1)\tau + \varkappa \int_0^\theta \Phi(\eta)\, d\eta - S(\theta)(1 - \tau) + F[\tau, \Phi, S, \delta] \right\} \qquad (1.11)$$

$$F[\tau, \Phi, S, \delta] = \delta(e^{-\tau} - 1 + \tau) - (e^{\tau} - 1 - \tau) +$$
$$\varkappa e^{-\tau} \int_0^\theta [e^{-\tau(\eta)} \sin\Phi(\eta) - \Phi(\eta)]\, d\eta -$$
$$\varkappa \int_0^\theta \Phi(\eta)\, d\eta + \varkappa e^{-\tau} \int_0^\theta \Phi(\eta) - S(\theta)(e^{-\tau} - 1 + \tau)$$

It is assumed here that with an accuracy to within the constant appearing in p_0'

$$p_0^*(x) = \sum_{n=1}^{\infty} \varepsilon^{n+2} d_n \cos \frac{2\pi n}{\lambda} x, \qquad S(\theta) = p_0^*[x(\theta)] \qquad (1.12)$$

where ε is a small positive dimensionless parameter and d_n are specified real numbers. The series $\Sigma \varepsilon^n d_n$ is convergent in the circle $\varepsilon_0 > 0$. To determine $S(\theta)$ it is necessary to substitute in (1.12) the values of $x(\theta) / \lambda$ obtained from the equation

$$\frac{x(\theta)}{\lambda} = -\frac{1}{2\pi} \int_0^\theta e^{-\tau(\eta)} \cos\Phi(\eta)\, d\eta \qquad (1.13)$$

which follows from (1.4).

Let us determine more accurately the formulas for parameters. In the case of a free wave $S(\theta) \equiv 0$ and it is necessary to set $c^2 = c_*^2(1-\varepsilon^2)$, where c_*^2 is defined by the following formula [1] for a free linear capillary-gravitational wave of length λ:

$$c_*^2 = \frac{2\pi\mu}{\lambda\rho} + \frac{g\lambda}{2\pi} \tag{1.14}$$

Taking into consideration the indicated expression for c, from (1.10) we obtain

$$\nu = \frac{\lambda\rho}{4\pi\mu} c_*^2(1-\varepsilon^2) = \nu^{(0)}(1-\varepsilon^2), \quad \nu^{(0)} = \frac{\lambda\rho c_*^2}{4\pi\mu} \tag{1.15}$$

$$\varkappa = \frac{g\lambda}{\pi c_*^2(1-\varepsilon^2)} = \varkappa_0\left(1 + \sum_{n=1}^{\infty}\varepsilon^{2n}\right), \quad \varkappa_0 = \frac{g\lambda}{\pi c_*^2}$$

Substitution of these expressions into (1.11) yields

$$\frac{d\Phi}{d\theta} = \nu^{(0)}\left\{\delta - 1 - (\delta+1)\tau + \varkappa_0\int_0^\theta \Phi(\eta)\,d\eta + \right. \tag{1.16}$$

$$\left.\varkappa_0\sum_{n=1}^{\infty}\varepsilon^{2n}\int_0^\theta \Phi(\eta)\,d\eta - S(\theta)(1-\tau) + F[\tau,\Phi,S,\delta]\right\} - \nu^{(0)}\varepsilon^2\{\ldots\}$$

where the expression omitted in the second braces must be the same as in the first braces.

Equality (1.16) defines the relation between functions $\tau(\theta)$ and $\Phi(\theta)$ at the circumference $|u| = 1$. The Dini relations [6]

$$\Phi(\theta) = \int_0^{2\pi} K_0(\eta,\theta)\frac{d\tau}{d\eta}\,d\eta, \quad K_0(\eta,\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\eta \sin n\theta}{n} \tag{1.17}$$

$$\tau(\theta) = -\int_0^{2\pi} K(\eta,\theta)\frac{d\Phi}{d\eta}\,d\eta, \quad K(\eta,\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\eta\cos n\theta}{n}$$

are valid for these.

We transform the terms in braces in (1.16), which are linear with respect to τ, Φ and ε, by using formulas (1.17) and integration by part. We then combine in the first braces the terms (with coefficients 2 and $-\varkappa_0$) with the same integrand $d\Phi/d\eta$ and with different kernels

$$K(\eta,\theta), \quad K_2(\eta,\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\eta\cos n\theta}{n^2}$$

where $K_2(\eta,\theta)$ is the first iteration of kernel $K(\eta,\theta)$.

The constants $\nu^{(0)}$ and $\varkappa_0$ in Eq. (1.16) are considered to be specified, since c_*^2 is fixed and δ is determined by the condition of periodicity $\Phi(\theta) = \Phi(\theta + 2\pi)$. Since ε appears in the right-hand part of Eq. (1.16), its solution and consequently also δ depend on ε. Let us set

$$\delta = \delta_0 + \delta'(\varepsilon) \tag{1.18}$$

The condition of periodicity at $\varepsilon \to 0$ implies that $\delta_0 = 1$, since for this the solution $\delta(\varepsilon)$ also tends to zero. After all transformations with allowance for (1.18), Eq. (1.16) assumes the final form (dots in the second braces denote the last six terms which are the same as those in the first braces)

$$\zeta(\theta) = \nu_1 \Big\{ \int_0^{2\pi} K^*(\eta, \theta)\, \zeta(\eta)\, d\eta + \delta'(\varepsilon) + \tag{1.19}$$

$$\delta'(\varepsilon) \int_0^{2\pi} K(\eta, \theta)\, \zeta(\eta)\, d\eta + \varkappa_0 \int_0^{2\pi} K_2(\eta, 0)\, \zeta(\eta)\, d\eta + \Psi(\theta, \varepsilon) \Big\} -$$

$$\nu_1 \varepsilon^2 \Big\{ 2 \int_0^{2\pi} K(\eta, \theta)\, \zeta(\eta)\, d\eta - \varkappa_0 \int_0^{2\pi} K_2(\eta, \theta)\, \zeta(\eta)\, d\eta + \ldots \Big\}$$

$$\zeta(\theta) = \frac{d\Phi}{d\theta}, \quad \Psi(\theta, \varepsilon) = \varkappa_0 \sum_{n=1}^{\infty} \varepsilon^{2n} \int_0^{\theta} \Phi(\eta)\, d\eta - S(\theta) \Big[1 + \int_0^{2\pi} K(\eta, \theta)\, \zeta(\eta)\, d\eta \Big] +$$

$$F[\tau, \Phi, S, 1 + \delta'(\varepsilon)]$$

$$K^*(\eta, \theta) = \sum_{n=1}^{\infty} \frac{\varphi_n(\eta)\, \varphi_n(\theta)}{\nu_n}, \quad \nu_n = \frac{n^2}{2n - \varkappa_0}, \quad \varphi_n(\theta) = \frac{\cos n\theta}{\sqrt{\pi}}$$

where ν_n are eigenvalues and $\varphi_n(\theta)$ eigenfunctions of kernel $K^*(\eta, \theta)$. It is also assumed that $\nu^{(0)} = \nu_1$, and parameter $\varkappa_0$ is selected so that the eigenvalue ν_1 is simple and positive [1]. Note that for $\nu^{(0)} = \nu_1$ parameter c_*^2 is determined by the specified formula, since (1.14) follows from formulas (1.19) for ν_n and from (1.15) for $\nu^{(0)}$ and $\varkappa_0$.

The condition of periodicity for function $\Phi(\theta)$ yields the relation

$$\delta'(\varepsilon) = -\varkappa_0 \int_0^{2\pi} K_2(\eta, 0)\, \zeta(\eta)\, d\eta + \tag{1.20}$$

$$\varepsilon^2 \Big[\delta'(\varepsilon) + \varkappa_0 \int_0^{2\pi} K_2(\eta, 0)\, \zeta(\eta)\, d\eta \Big] - \frac{1 - \varepsilon^2}{2\pi} \int_0^{2\pi} \Psi(\theta, \varepsilon)\, d\theta$$

The problem has been thus reduced to the determination of functions $\zeta(\theta, \varepsilon) = d\Phi / d\theta$ and $x(\theta, \varepsilon) / \lambda$ and of the constant $\delta = 1 + \delta'(\varepsilon)$ from the system of Eqs. (1.13), (1.19) and (1.20), and $\tau(\theta, \varepsilon)$ determined by (1.17) and

$$\Phi(\theta, \varepsilon) = \int_0^{\theta} \zeta(\eta, \varepsilon)\, d\eta \tag{1.21}$$

Using (1.13) for eliminating $x(\theta, \varepsilon) / \lambda$ in Eqs. (1.19) and (1.20) and taking $\tau(\theta, \varepsilon)$ and $\Phi(\theta, \varepsilon)$ from (1.17) and (1.21), respectively, we reduce the system to two equations, viz.: (1.19) and (1.20). Equation (1.19) is nonlinear integral with respect to $\zeta(\theta, \varepsilon)$ with kernel $K^*(\eta, \theta)$ and parameter $\nu^{(0)} = \nu_1$. Equation (1.20) is nonlinear and transcendental with respect to the constant $\delta'(\varepsilon)$. However it is more convenient to consider the system of three equations without resorting to the above transformation. Then the only integral equation which is nonlinear with respect to $\zeta(\theta, \varepsilon)$ is Eq. (1.19); the remaining equations as well as (1.19) must be considered transcendental and nonlinear with respect to $x(\theta, \varepsilon) / \lambda$ and $\delta'(\varepsilon)$ with the operators and functionals linear with respect to the unknown functions.

2. Solution of basic equations of the problem. We seek the solution of the system of Eqs. (1.13), (1.19) and (1.20) in the form of series in powers of parameter ε. For each of the coefficients of the expansion of function $\zeta(\theta, \varepsilon)$ we obtain a linear integral Fredholm equation of the second kind with kernel $K^*(\eta, \theta)$ and parameter $\nu^{(0)} = \nu_1$ (ν_1 is the first eigenvalue of that kernel). For the first coefficient of that expansion we obtain a homogeneous integral equation which is solved by the second Fredholm theorem. Equations for the coefficients of all subsequent approximations are nonhomogeneous, and are solved by the third Fredholm theorem. The solution of each of these equations is in the form of a sum of solution of a homogeneous equation with undetermined coefficient C_{1n} (for the n-th approximation) and of the particular solution of the nonhomogeneous integral equation. The coefficient C_{1n} is determined by the condition of the equation solvability in the $(n+2)$-nd approximation.

Thus each of coefficients C_{11}, C_{12} and C_{13} is determined by the condition of solvability of equations in the fourth, fifth and sixth approximations.

For the coefficients of expansions of remaining quantities we obtain a system of linear algebraic equations. That system, which is always solvable, yields for the coefficients of a given approximation explicit expressions in terms of quantities determined in preceding approximations.

2.1. Determination of the first three approximations. Below we present third approximation expressions for $\zeta(\theta, \varepsilon)$, $x(\theta, \varepsilon) / \lambda$ and $\delta'(\varepsilon)$

$$\zeta(\theta, \varepsilon) = \varepsilon C_{11} \cos\theta + \varepsilon^2 C_{22} \cos 2\theta + \varepsilon^3 (C_{13} \cos\theta + C_{33} \cos 3\theta) \tag{2.1}$$

$$\frac{x(\theta, \varepsilon)}{\lambda} = -\frac{\varepsilon}{2\pi} C_{11} \sin\theta - \frac{\varepsilon^2}{2\pi}(C_{11}^2 + C_{22}) \sin 2\theta - \frac{\varepsilon^3}{2\pi}\left[C_{13}\sin\theta + \frac{1}{3}\left(\frac{1}{6}C_{11}^3 + \frac{1}{2}C_{11}C_{22} + \frac{1}{3}C_{33}\right)\sin 3\theta\right]$$

$$\delta'(\varepsilon) = -\varepsilon \varkappa_0 C_{11} + \frac{\varepsilon^2}{4}\varkappa_0 (C_{11}^2 - C_{22}) - \varepsilon^3 \varkappa_0 \left(\frac{1}{18}C_{11}^3 + \frac{1}{6}C_{11}C_{22} + C_{11} + C_{13} + \frac{1}{9}C_{33}\right)$$

where

$$C_{12} = 0 \text{ (see (2.5))}, \quad C_{22} = -{}^3/_4\, \varkappa\, C_{11}^2 \nu_1 \nu_2 / (\nu_2 - \nu_1) \tag{2.2}$$

$$C_{33} = {}^1/_{12} C_{11} [C_{11}^2 (1 - {}^{11}/_3 \varkappa_0) - {}^{13}/_2\, \varkappa_0 C_{22}]\, \nu_1 \nu_3 / (\nu_3 - \nu_1)$$

Coefficient C_{13} has not been calculated, because the fifth approximation required for its determination was not computed; C_{11} is determined by the equation

$$[{}^1/_4 + {}^9/_{32}\, \varkappa_0^2 \nu_1 \nu_2 / (\nu_2 - \nu_1)]\, C_{11}^3 - 2C_{11} - d_1 = 0 \tag{2.3}$$

Note that for $d_1 = 0$ Eq. (2.3) is the equation which determines C_{11} in the case of a free wave.

2.2. Determination of further approximations. As previously stated, the coefficient C_{12} is determined by the solvability condition of the equation for $\zeta_4(\theta)$, which leads to the following relation:

$$C_{12} C_{11}^2 [1 + {}^9/_8 \varkappa_0^2 \nu_1 \nu_2 / (\nu_2 - \nu_1)] = 0 \tag{2.4}$$

Since $C_{11} \neq 0$ and, as can be shown, the expression in brackets is also nonzero, hence $C_{12} = 0$

It can be shown by the method of mathematical induction that, as in the case of $n = 3$, $\zeta_n(\theta)$, $x_n(\theta)$ and δ_n are similarly uniquely determined for any positive integral $n \geqslant 3$. The equation for C_{1n} is linear beginning with $n = 2$, and the coefficient at C_{1n} is the same as in (2.4).

3. Determination of the wave profile. The wave profile in parametric form $x(\theta, \varepsilon)$ and $y(\theta, \varepsilon)$ is determined by formulas (1.4) into which it is necessary to substitute $\Phi(\theta, \varepsilon)$ and $\tau(\theta, \varepsilon)$. We recall that functions $\tau(\theta, \varepsilon)$ and $\Phi(\theta, \varepsilon)$ are determined in terms of $\zeta(\theta, \varepsilon)$ by formulas (1.17) and (1.21), respectively. Eliminating θ in the parametric equations, we obtain for the profile an equation of the form $y = y(x, \varepsilon)$.

The equation of the wave profile, accurate to within third order terms, is

$$y(x, \varepsilon) = k^{-1}\{\varepsilon C_{11}(\cos kx - 1) + {}^1/_4\, \varepsilon^2 (C_{11}^2 - C_{22})(1 - \cos 2kx) + {}^1/_6\, \varepsilon^3 [(6C_{13} + {}^9/_4\, C_{11}C_{22})(\cos kx - 1) + ({}^1/_3 C_{11}^3 - {}^5/_4\, C_{11}C_{22} + {}^2/_3\, C_{33})(\cos 3kx - 1)]\} \qquad (3.1)$$

where $k = 2\pi / \lambda$ and coefficients C_{ij} are defined by formulas (2.2) and (2.3).

Note. As stated in Sect. 1, the coordinate origin is located at the crest of the wave, hence for x close to zero y must be negative. Analysis of the principal term in (3.1) shows that $C_{11} > 0$, and Eq. (2.3) implies that to satisfy this condition it is necessary to set $d_1 > 0$.

4. Existence and uniqueness of solution of the problem. Using the Liapunov-Schmidt methods and their development [7], we can establish the following theorem.

Theorem. The system of Eqs. (1.13), (1.19) and (1.20) has the unique solution $\zeta(\theta, \varepsilon)$, $x(\theta, \varepsilon) / \lambda$ and $\delta'(\varepsilon)$ $(\delta'(\varepsilon) = \delta(\varepsilon) - 1)$ which is small with respect to ε and continuous with respect to θ $(0 \leqslant \theta \leqslant 2\pi)$ and that solution is an analytic function of ε for small $|\varepsilon| < \varepsilon_1 \leqslant \varepsilon_0$.

The proof of the theorem is similar to that presented in [8, 9].

The absolute and uniform convergence of series for $\Phi(\theta, \varepsilon)$ and $\tau(\theta, \varepsilon)$ follows from the theorem. The convergence of series in powers of ε for the integrand functions in (1.4) follows from general theorems on the analysis of the substitution of series into series. The convergence of the series, whose approximate sum is defined by formula (3.1), is established on the basis of general theorems of analysis.

Note For solving this problem function $p_0^*(x)$ was specified in the form (1.12), which made it possible to derive the solution in the form of series in integral powers of parameter ε. If it is assumes that

$$p_0^*(x) = \sum_{n=1}^{\infty} \varepsilon^n d_n \cos \frac{2\pi n}{\lambda} x$$

then it is possible to show by analyzing the branching equation of the Liapunov-Schmidt method that it would be necessary to construct the solution in the form of series in powers of $\varepsilon^{1/3}$.

REFERENCES

1. Sekerzh-Zen'kovich, Ia. I., On a form of steady capillary-gravitational waves of finite amplitude. PMM Vol. 34, №6, 1970.
2. Sekerzh-Zen'kovich, Ia. I., On stationary capillary-gravitational waves of finite amplitude induced by pressure periodically distributed over the surface of a fluid stream of infinite depth. Dokl. Akad. Nauk SSSR, Vol. 195, №2, 1970.
3. Sekerzh-Zen'kovich, Ia. I., On the theory of stationary capillary-gravitational waves of finite amplitude. Dokl. Akad. Nauk SSSR, Vol. 109, №5, 1956.
4. Sekerzh-Zen'kovich, Ia. I., Steady cappilary-gravitational waves of finite amplitude at the surface of an infinitely deep fluid. Tr. Morsk. Gidrofiz. Inst. Akad. Nauk UkrSSR, Vol. 27, 1963.
5. Sekerzh-Zen'kovich, Ia. I., On composite steady gravitational waves of finite amplitude. PMM Vol. 33, №4, 1969.
6. Nekrasov, A. I., The Exact Theory of Waves of Stabilized Form at the Surface of a Heavy Fluid. Izd. Akad. Nauk SSSR, Moscow, 1951.
7. Vainberg, M. M. and Trenogin, V. A., Methods of Liapunov and Schmidt in the theory of nonlinear equations and their further development. Uspekhi Matem. Nauk, Vol. 17, №2 (104), 1962.
8. Sekerzh-Zen'kovich, Ia. I., On a form of steady waves of finite amplitude. PMM Vol. 32, №6, 1968.
9. Sekerzh-Zen'kovich, Ia. I., On a particular form of steady waves. Book: The Theory of Solution Branching of Nonlinear Equations (in Russian). "Nauka", Moscow, 1969.

Translated by J. J. D.

UDC 541.124:532.5

ASYMPTOTIC ANALYSIS OF STATIONARY PROPAGATION OF THE FRONT OF PARALLEL EXOTHERMIC REACTION

PMM Vol. 39, №2, 1975, pp. 306-315

V. S. BERMAN and Iu. S. RIAZANTSEV
(Moscow)

(Received July 19, 1974)

We develop an approximate theory of stationary propagation of the planar front of a two-stage parallel exothermic reaction in a condensed medium and in a gas. In constructing the solutions we use the method of matched asymptotic expansions. As parameter of the expansion we employ the ratio of the sum of the activation energies of the reactions to the terminal temperature, the latter being determined in the course of solution of the problem. We show the characteristic limiting modes corresponding to the various parameter values which appear in the problem. For each of these modes we obtain approximate analytical expressions for the wave velocity, the distribution of concentrations, and the terminal temperature.

1. Statement of the problem. The stationary propagation of the planar